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MATHEMATICS FOR THE JUNIOR HIGH SCHOOL.*

BY GEORGE W. EVANS.

We have had a committee of the Association of Teachers of Mathematics in New England appointed to deal with the criticisms brought against the teaching of mathematics in secondary schools. This attack has been vigorous, and in many cases thoughtful.

Our critics have pointed out that the teaching of mathematics in secondary schools, and particularly the teaching of algebra, has been remote in its nature from the interests of the pupils of that age; that it has had little content suitable for application in the immediate present; and that the prescriptive character of the mathematics program served to exclude subjects that might well be thought more desirable for pupils who were not to go to college, and for those that were likely to be seriously considering an early necessity for earning their living.

Our critics have suggested that in any reconstruction of the mathematics course there should be included a year's work of such comprehensive character that the pupil might obtain from it a general idea of what mathematics was about, so that his further pursuit of the subject could be grounded upon a more or less intelligent appreciation.

Our committee has dealt with several phases of the subject; and what I propose to speak of to-day is a tentative plan for reconstruction which is intended to meet the suggestion of our

* Read at the spring meeting of the Association.

friends the enemy. We have had the advantage of informal cooperation with a committee of public-school teachers in Boston who have been asked by the school authorities to propose a mathematics program for a so-called junior high school. This, as you know, is intended as a reorganization of the seventh, eighth and ninth grades into a school intermediate between the elementary school and the secondary school. One of the topics of the reconstructed course has been approximate computation. You are familiar with it. It involves the rejection of superfluous numbers in the course of the computation, and not merely at the end: the estimation of the accuracy of data and of results by the number of significant figures rather than by the number of decimal places; and the systematic teaching of such checks as can be used without the retention of all figures. This retention would, of course, be necessary in using such checks as casting out nines.

We have had some difficulty in persuading teachers of the subject to begin multiplying from the left-hand end of the multiplier. The current method, by which we begin at the right-hand end of the multiplier, was originally an alternate to the one that we are now recommending, and was fixed upon in the usage of schools for no good reason that I can discover. Multiplying from the left-hand end was a companion of the early use of decimal fractions at the beginning of the seventeenth century; but since this was very soon followed by the invention of logarithms, it was probably felt that approximate computation could be left to the man with the logarithm table. At any rate, the wrong way was, up to the beginning of the present century, pretty firmly intrenched in the school system.

Very curiously, the most conservative country in the world, England, has within the last fifteen years completely changed in this important respect. When John Perry proposed the reform at the Glasgow meeting in 1901 (or 1902—I forget which), multiplying from the right-hand end was so fixed a custom that he suggested re-writing the multiplier wrong end to, so that the computer could feel that he was adhering to his grandfather's ways. The custom now seems to be so universal to follow the other order that a questionnaire in arithmetic issued by the Mathematics Association in England proposed several methods

of fixing the decimal point, and in all of them the reverse order of multiplication (it should really be called the natural order) is given without apparent alternative, quite as a matter of course.

In our proposed reconstruction, this is led up to through the other grades, so that in the ninth grade, for the first time, the pupil gets the complete system of approximate computation, including the rejection of figures in the course of his work.

Other topics include algebra and some geometry. Algebra it is proposed to introduce by means of formulæ which are presented as systematic abbreviations of arithmetical rules, and problems which the pupil is told he must still use arithmetic for. He can, however, systematically abbreviate the explanation of these problems in such a way that, having put down the reasoning in successive steps, he shall soon be enabled to follow the explanation of a problem that was previously quite beyond his reach. Gradually increasing algebraic difficulties are introduced by new problems; the pupil having in mind always that his algebra is for the purpose of solving these problems. Problems selected for this use will not conform to the requirement of immediate practical application. In this connection I should like to refer you to Mr. Carson's book on the teaching of mathematics, published by Ginn & Company last year. What he has to say about real problems is an interesting reply to the criticisms that are now fashionable. It may be noted, also, that the practical problems in arithmetic used almost exclusively in elementary schools are those that refer to the counting room. Problems that have to do with measurements have been much neglected.

Mr. Carson's book has also much to say about the proofs usually given at the beginning of instruction in geometry.

The program that we have been considering includes a dozen or more of the most important propositions in elementary geometry, including the Pythagorean theorem, measurement theorems, and the principle of similarity. Proofs have been prepared for these propositions which our teachers believe do not unnecessarily insist upon what a mathematician would call rigor. These proofs aim rather to satisfy the doubts of pupils in regard to the necessary and inevitable character of the inferences that they are called upon to make. Truths that they accept

without demonstration are not encumbered with demonstration; nevertheless, enough demonstration is included to enable the pupil to get the idea of successive logical dependence, which is really the main lesson of elementary geometry teaching.

It is proposed to begin in Boston a year from next September some twenty of these junior high school units, in different sections of the city, where the conditions of the school organization seem to warrant the change. Our greatest difficulty will be, as always, the lack of text-books, and the difficulty of co-ordinating new material in a system so completely organized. We hope to have enthusiastic co-operation on the part of a great many teachers who have already expressed a warm interest in the proposition.

CHARLESTOWN HIGH SCHOOL,
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EUCLID'S FALSE AXIOM.

BY WILLIAM A. FRANCIS.

As youth gives place to creeping age.
One by one our idols fall;
We read anew life's blotted page;
Our gods are gone for good and all.

In youth I read in weighty tome:
"Hairs of thy head all numbered are."
With age the message sad comes home:
"The numbers go not very far."

In youth I read on Euclid's page:
"The whole is greater than the part."
I cried: "How true, O wondrous sage!
This is a fact that stirs the heart!"

With age my hair is wholly gone;
I view with pain a polished poll.
I find false Euclid quite forsworn;
The part is equal to the whole.

WHAT IS TO BE THE OUTCOME?*

By David Eugene Smith.

1. Recent experience in certain parts of the country shows that it is not unlikely that, within a few years, students may rather generally, at any rate for a time, be admitted to college without mathematics. This movement is intended to be democratic in its purpose, but it may well be doubted whether it is not more likely to turn out to be a phase of the return to the old aristocracy of education. By depriving students of equal privileges in the way of serious mental work, the danger is the same as that of placing the extremely difficult subject of vocational guidance in the hands of any except a highly trained, highly gifted, and carefully selected group of experts—that of depriving all of equal chance in the race of life. To reduce this danger to a minimum, it is essential that we carefully consider, in meetings of this nature, our attitude in the face of the attacks now being made upon mathematics.

2. The attack upon mathematics in the secondary school has been of two natures:

(*a*) The attack of educators whose powers of speaking and writing lead them to court applause through their cleverness of statement. Men of this type are found in various institutions, and our experience shows that their influence is ephemeral.

(*b*) The attack of more serious scholars who point out the defects in our scheme of mathematics and suggest remedies. These scholars are often found outside our guild, but much more commonly in associations like this which I address. Their purpose being constructive, their influence is certain to be felt.

3. The attack upon mathematics on the ground that it has no general disciplinary value has thus far been abortive, scientifically. We have only to see how divergent are the results of various investigations to see the truth of this assertion. There has been not the slightest proof adduced that mental exercise requiring vigor of thought does not conduce to brain vigor,

* Read at the spring meeting of the Association.

exactly as physical exercise conduces to bodily vigor. What has been shown is that certain specific claims, rarely made for many years past, seem not to be valid.

4. Whatever may be the future of mathematics, the science will continue to be taught in the secondary schools for many years to come much as it is at present, because of the mere force of inertia if for no other reason. Schools change slowly, and the training of the necessary teachers can proceed only at a certain rate.

5. Referring only to the general type of schools, it would seem that we are tending to a constructive reform from within, somewhat along the following lines:

(*a*) A six-year high school.

(*b*) The first three years will probably offer required courses in mathematics, and the second three years will offer elective courses.

(*c*) The three years of required mathematics, covering the seventh, eighth, and ninth school years, will include applied arithmetic, intuitional and constructive geometry, the uses of algebra, and an introduction to demonstrative geometry and formal algebra. These years will open the door of mathematics, will show its general nature, and will determine whether a student has the mental equipment for the further study of the subject.

(*d*) The three years of elective mathematics, covering the tenth, eleventh, and twelfth school years, will include the algebra necessary for a later study of college mathematics, a reduced number of basal propositions of plane and solid geometry, with plenty of exercises, the essential features of plane trigonometry, arithmetical calculations with logarithms and the slide rule, mechanics or applied mathematics, shop drawings, and the first notions of differentiation and integration.

All this is a mere commonplace in the best European schools, and the outlook is in favor of our adopting the best features there found.

If we can set about to do this work with the same attention to scientific arrangement of matter that characterized our earlier courses, avoiding the pitfalls of a mere notional and unscientific sequence, we shall secure better mathematics than at present,

studied with more interest, and at the same time we shall secure a mathematics that is more closely in touch with present life.

6. With this constructive program, there seems little to fear from the present attacks on mathematics. Even aside from the value of the subject per se, which seems somewhat a matter of fashion rather than of scientific decision at present, it does not stand on the same foundation as the classics, since the practical applications of mathematics are multifarious, and we have only to show the lines of these applications to secure a position that is not likely to yield to any such assaults as those which are now being made. Certainly, as compared with any material which is now being offered to replace it, even the classics should have a strong position, but the position of mathematics should be impregnable.

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OUR CRITICS AND THEIR VIEWPOINTS.*

By J. H. Minnick.

The purpose of this paper is to present the viewpoints of those who criticize present practices in the teaching of mathematics. It has been impossible to make an exhaustive study of the criticism against mathematics. This discussion is therefore limited to the work of a few men whose viewpoints, it is believed, may be considered as representative. We shall consider representatives from three classes of school men; namely, university professors, state superintendents, and men active in local school systems. The facts have been gathered from letters written by critics in response to our inquiries, and from their published works. We shall try to give a fair statement of these facts free from the influence of our personal opinion.

Professor Frank M. McMurry, of Teachers College, has expressed his views concerning mathematics in the *Teachers College Record* of March, 1903.† "Business utility and mental discipline," he says, "should not rank as the primary aims in teaching mathematics. . . . The child's interest in the quantitative side of life should be the highest immediate aim of the teacher of mathematics in the grades. . . ." The subject matter should be selected in accordance with the child's nature and with a view to meeting social needs. As we understand Professor McMurry, mathematics is to be used as a means for the quantitative interpretation of life and as such must be taught in close relation to the affairs of life. In answer to our letter of inquiry Professor McMurry says: "I have frequently suggested criticism of mathematics on the grounds that it deals with the processes, without reference to the purposes of those processes. For example, outside of educational institutions people continually work problems for the sake of getting the answers. In arithmetic and algebra, however, the student is expected to dis-

* Read at the spring meeting of the Association.

† The chapter "Controlling Ideas Throughout the Curriculum" was written chiefly by Professor McMurry.

regard the answer, practically, even though working for it. That situation seems remarkably artificial."

In his letter replying to our inquiry, Professor W. H. Kilpatrick makes three points clear: (1) he would not justify mathematics in secondary education only on the basis of formal discipline; (2) he would differentiate the courses in mathematics to meet the needs of various groups of children, and (3) he believes that the amount of mathematics should be decidedly less than is now taught in the secondary schools of our country. He says: ". . . if we follow the logic of our present thinking along the line of formal discipline, it seems to me, then, that we are compelled to question seriously the traditional high-school mathematics. I cannot see for my own part why we need one tenth as much of the geometry as is offered, except possibly for those who are specializing in mathematics for research purposes. It further seems to me that if we consider the needs of the various groups of users, we must give up the notion of teaching algebra in one form to all."

A severe criticism against present practices in mathematics is offered by Henry C. Morrison, superintendent of public instruction of New Hampshire in an article entitled "Reconstructed Mathematics in the High School" and published in The Thirteenth Yearbook, Part I., of the National Society for the Study of Education. He maintains that there is general dissatisfaction with the results of mathematical instruction. He then attempts to analyze the situation with a view to determining the cause of the failure of mathematics and the remedy for this failure. He finds three factors in the educational problem; namely, the child, social needs, and the subject-matter by means of which the child can be adjusted to these social needs. Mathematics, he maintains, has failed for two reasons. First, it can not be made to function in the child's life. "It not only does not function in the hands of a poor teacher," says he, "but it cannot even in the hands of the skilled teacher. That is to say, there is nothing to which most processes in algebra or geometry, or indeed arithmetic, can be applied except to more algebra or arithmetic." Nevertheless Mr. Morrison suggests an array of mathematical subjects such as the teacher of mathematics would scarcely dare propose in the face of present-day criticism. For

the domestic science course he suggests arithmetic and mechanical drawing; for the agricultural course, geometry, trigonometry and algebra; for the commercial course, science of accounts, higher arithmetic and algebra; and for the mechanic arts course he suggests geometry, plane trigonometry, analytic geometry and calculus. It is difficult to see why such subjects should be included in a high-school course if the present course has failed because mathematics can not be made to function "even in the hands of the skilled teacher." The second cause of failure is the fact that our mathematics is not suited to the nature of the adolescent. It seems that his objection here is to drill work as well as to the formal side of mathematics. He declares that the period of adolescence "is the worst period between the shedding of milk teeth and the grave for anything like drill."

Mr. Morrison offers a remedy for the failure of mathematics. "In the first place," he says, "the traditional round of algebra, geometrical logic, advanced algebra, and trigonometry ought to be entirely abandoned and a fresh start made." Having thus cleared the board, he would begin the differentiation of courses with the seventh grade, this differentiation to provide courses in household arts, agriculture, mechanic arts, commerce, etc. Each course is to have its own specially organized mathematics including those elements essential to the work of that course. Further the mathematics of each course should probably be taught by the teachers in charge of the course rather than by a special mathematics faculty.

Another criticism of interest comes from Dr. Snedden, commissioner of the Massachusetts State Board of Education. In a letter* to the committee appointed by the Association of Mathematical Teachers in New England to investigate current criticism on high-school mathematics Dr. Snedden proposes the following questions:

"1. What are, or what should be, the controlling purposes to be served by the study of algebra in the secondary schools? . . .

"2. Is it yet practicable to demonstrate either that these purposes are important for all pupils, or that they are realized in the case of many of the pupils taking the study?

* See Preliminary Report of the Committee on the Status of Mathematics in the Secondary Schools.

"3. Are these purposes as yet so definitely formulated that it is practicable to determine how far given organizations of the materials of algebra, or given methods of teaching, results in their realization?

"4. More specifically, why should girl students in the high schools be required to take algebra before being allowed to graduate?

"5. Why should women students seeking admission to college be required to present algebra as an entrance subject?"

From these questions it seems that Dr. Snedden doubts whether, in case the purpose of mathematics were clearly defined, that purpose would be of great value to all students or whether it is realized in the case of many students. Further he believes that the prominent place now held by mathematics in the secondary schools is due to faith in "mental training" rather than its usefulness in further learning. He makes the following suggestions for a high-school course: (1) Make mathematics an elective both for graduation from the high school and for admission to college, (2) offer a course in mathematics for those students preparing for vocations which require the subject, and (3) develop a cultural course in mathematics for those who wish to inform themselves more fully about the world in which they live. "Just as many of us," he says, "can enjoy and respond to operas, epics, and great paintings, without being artists in these fields, so I think many could be led to appreciate the place of mathematics without becoming mathematicians." We leave each to draw his own conclusions concerning such a course but to us it seems to be a course in mathematics with the mathematics left out.

Associate Superintendent William McAndrew,* of the City of New York, has undertaken to investigate the results of the teaching of mathematics in the grades of the Brooklyn schools. His investigation shows that even in simple problems involving only the fundamental processes of arithmetic, accuracy has not been acquired. He shows further that with small expenditure of time in drill the efficiency of children can be greatly increased. His chief criticism is directed against the tendency to stress the process and speed at the expense of accuracy.

* See Report upon Division 4 and 5, Elementary Schools. Department of Education, The City of New York.

William D. Lewis, in his "Democracy's High School," declares that "high-school mathematics is limited by college entrance requirements almost entirely to abstract theory and manipulative gymnastics." He claims that this course is a failure and as proof he cites the fact that the report of the New York State Education Department for 1913 shows that only 67.1 per cent. of the pupils taking the examination in mathematics passed. We note, only as a passing remark, that when judged by the same standards Italian, Latin, science and the commercial subjects are even worse failures. Mr. Lewis suggests that (1) mathematics be brought in closer relation to real life, (2) pure mathematics be reserved for the colleges, (3) more attention should be given to speed and to the accuracy of results. In answer to our letter he says: "My objection to the traditional requirements in mathematics is largely empirical. I have seen so many pupils driven out of school by work which could not by any possibility be of practical advantage to them, and I have watched so many classes in guessing, under the caption of algebra, that I have come to believe that we ought to discriminate as carefully as possible between those pupils who really need the advanced mathematics and those who will find other work more profitable."

In general our critics are agreed "that mental training is not sufficient reason for giving mathematics its present importance in the curriculum, that it should be more closely related to the affairs of life, and that the subject-matter should be better suited to the nature and needs of the pupils, which generally means that the subject should be less formal and that the course should be differentiated. However they are not fully agreed as to the extent of the mathematics which should be taught nor are they agreed as to the methods by which the purpose of mathematics is to be realized.

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THE ESTABLISHED RESULTS OF THE NEW PSYCHOLOGY AS IT BEARS UPON THE TEACHING OF MATHEMATICS.

By Agnes L. Rogers.

By the New Psychology we indicate the comparatively modern psychology based upon experiment and measurement. Its results have been obtained from investigations, which have been conducted under controlled conditions and which can therefore be repeated by other investigators and verified. The older analytical psychology, which used the method of introspection, made valuable contributions to our knowledge of the structure of mental processes; but from a practical point of view it is more important to know their dynamical character and functional relations and this is particularly true as regards teaching, since the fundamental problem for education is "the production and prevention of changes in human beings."

Psychologists have made, however, relatively few experimental investigations in the field of mathematics. If we disregard arithmetic, which has of late received considerable attention from the experimentalists, we find that most of the publications upon the psychology of the subject have used the older methods of introspection and observation. This is not surprising, when we recall how little is known as yet of the psychology of thought and how indispensable systematic observation is as a preliminary to experimentation. The result is that we have at present various theories of a speculative character as to the nature of mathematical ability, which still await confirmation. Möbius, for example, has advanced the view that mathematical talent is a special capacity, independent of other mental capacities and characterized by unusual ability in understanding relations of number, in judging relations of size and in concrete imagery. Hüther, on the other hand, maintains that mathematical genius involves no specific, fundamental capacity; it consists merely in an exceptional ease in carrying out certain thought operations. It involves marked development of con-

crete imagery, synthetic imagination, and mathematical understanding. Betz, who agrees with Hüther's general theory, offers a different explanation of mathematical ability. He contends that the mathematical type of mind is characterized by a special clearness of certain "minimal" or highly abstract ideas, and by the ability to manipulate and vary these with ease and precision. Again, Henri Poincaré, the distinguished mathematician, on the basis of introspective analysis gives it as his judgment that mathematical ability has nothing to do with a very sure memory or a special power of attention. It is a feeling for order and the concealed relations of numbers that distinguishes the mathematicians from other men. He divides mathematical reasoners, however, into two distinct classes, the geometrical or intuitional and the analytical or logical types.

All such theories are interesting, but singularly barren of fruitful practical suggestions. At best they merely indicate to the experimental psychologists promising subjects of research. It should not be inferred, however, that experimentation alone can furnish information of value to the mathematics teacher. On the the contrary, such a method of attack as that of Judd furnishes an illuminating account of the mental activities that mathematics demands. It presents an excellent survey of the simpler psychological processes underlying mathematics, which have been experimentally investigated, describing typical mental reactions involved in mathematical thinking and analyzing the psychological implications of the text-books in use and of current class-room procedure. In comparison, the New Psychology has little to offer; but by its refinement of method it undoubtedly gives promise of richer results.

Early experimental effort was naturally directed to analysis of the mental processes involved in the simplest branches of the subject, namely arithmetical operations. The work in this field has been both extensive and significant for mathematics in general. For our purpose the most important results of the statistical studies by Rice, Thorndike, Stone, Bonser, Courtis, Winch, and Starch* are the demonstration of the wide range of individual differences in capacity and the evidence in support of the specialization and independence of the different abilities involved

* See Bibliography, H. B. Howell, "A Foundational Study in the Pedagogy of Arithmetic."

in arithmetic. The extent of individual differences had already been shown by investigations upon other mental functions; its demonstration in the case of arithmetic, however, was exceptionally striking. Equally remarkable was the discovery that a high degree of excellence in the fundamental processes may be present along with a low degree of skill in arithmetical reasoning and vice versa. Indeed, it was found that a similar variability prevailed among the fundamental processes themselves. These results led Fox and Thorndike to prophesy that the abilities tested—addition, multiplication, fractions, rational computation and problems—bear little resemblance to those of the mathematician.

The amount of experimental investigation accomplished in algebra and geometry falls far short of the work done in arithmetic. If we exclude the recent efforts to establish standards for algebra by Thorndike, Monroe, and Rugg, a new line of activity, which is bound to have considerable effect upon the teaching of algebra, we find that in all the experimental investigations published, with two exceptions, the data have been school and college marks or class lists. Correlations have been calculated between mathematical ability as a whole and ability in other school subjects† and in brief the result has been that fairly high positive correlations have been obtained.

An interesting attempt to secure a more complete and detailed analysis of mathematical intelligence was made in 1910 by William Brown. He used the same statistical method of correlations, obtaining his data from a school examination in algebra, geometry and arithmetic. He corrected the papers, however, by two methods, first according to ordinary school standards, and second according to a differential system of marking, based on an introspective analysis of the intellectual processes involved in answering. His principal results are, first, that geometrical ability and algebraic ability are not related, save through their connection with arithmetical ability, which is of some interest with reference to the present effort to correlate these subjects more closely, and second, that memory of preceding propositions is the central ability in geometry, being related most intimately to other forms of geometrical ability. This is in harmony with

† See Columbia Contributions to Philosophy, Vol. XI., No. 2.

his opinion that school mathematics and higher mathematics relate to different forms of ability and it raises the whole question of the distinction between school mathematics as it *is*, and as it *might be*. The fact that it is now advocated in some quarters that the concepts of higher mathematics should be introduced into the secondary school suggests that even if Brown's conclusion is true of the present state of affairs, it need not be with different methods of teaching and other standards. In any event his experimental procedure should lead us to accept his statements with caution; for obviously the "psychologizing" of examination papers is an unsatisfactory manner of measurement, and further the number of individuals examined was relatively small.

Another statistical study carried out in the Dartmouth pedagogical department under the direction of F. C. Lewis, deserves mention on account of the new departure in method. Instead of using ordinary school marks as data, tests were given in originals in geometry and in practical reasoning and the scores made in these were correlated. It may be doubted whether these tests were adequate measures of the abilities in question; but the mode of procedure certainly marks a step in advance and the results are noteworthy. The pupils of each of twenty-four groups were arranged in two series, the first according to their ranking in mathematical reasoning and the second according to their ranking in practical reasoning. It was found that of the first five mathematical reasoners from each group, 63 per cent. were at the foot of the practical reasoning series, conspicuous for their inefficiency in practical reasoning, and of the pupils at the foot of the mathematical reasoning series, 47 per cent. were conspicuous for their positions at the head of the practical reasoning series.

These statistical inquiries like the earlier accounts based on introspection and observation are interesting rather than helpful. Their general outcome can be briefly expressed in the statement that *a high correlation exists between efficiency in mathematics and efficiency in other subjects.*

For further guidance we must turn to general psychology and here the crucial question is transfer of training and for two reasons. Not only do experiments upon transfer yield the most

useful suggestions for methods of teaching; upon them ultimately rests the defence of the place accorded to mathematics in the curriculum of the secondary school. For the mathematicians themselves have admitted that all the facts that a skilled mechanic or engineer would ever need could be taught in a few lessons. Consequently very little of the mathematics at present given in high school could be retained on the ground of its practical usefulness. Nor does the conventional value of mathematics justify the time and effort it entails. Society does not regard an individual as grossly ignorant or ill-informed, if his knowledge of mathematics is extremely meager. Current opinion rather assumes that mathematical skill is highly specialized and unrelated to general intelligence or culture. The ultimate defence of the retention of mathematics in the curriculum rests therefore upon its general educational value.

What then are the established results as regards transfer of training? *At the present time no psychologist of repute denies that there is transfer.* Experimentation has conclusively shown that practice in one function affects other functions. The points at issue are the extent to which transfer takes place and the methods by which it is secured. *It has been established that the amount of transfer varies with the degree of difference between the functions in question.* Change either in the content or in the method of study reduces the extent of the spread of improvement. Accordingly the indirect effect of practice is invariably less than the direct. Furthermore transfer can be negative: the habits or mental acts developed by a particular kind of training may inhibit rather than facilitate other mental activities. Investigators have found very different degrees of transfer effect in accordance with the different functions tested and the different experimental conditions and frequently it has been extremely small in amount; but on the whole there is ground for the statement that if transfer is often not largely general, it is probably always to some extent general and "a very small spread of training may be of great educational value, provided it covers a wide enough field." As Thorndike points out, "*if a hundred hours of training in being scientific about chemistry produces only one hundredth as much improvement in being scientific about all sorts of facts,*

it would be a very remunerative educational force." Certain facts have also to be borne in mind in reference to the smallness of the transfer obtained in experimental investigations. For the most part the latter have used adults as subjects and in their case we would not expect to obtain as much transfer as in the case of children. Whereas in the young and immature mental habits are still in process of formation, in adults these are practically established and in consequence any improvement made in training is probably due to adaptation to special conditions and therefore not susceptible of generalization. The results of Dallenbach's experiments on visual apprehension in school children contrast strongly with those of Whipple and Foster on adults as regards practice effects and cast doubt upon the common assumption that conclusions derived from a limited number of selected adults are necessarily true of growing children. Again, certain features of the experiments made on transfer seem distinctly unfavorable to spread of improvement. Thus the practice periods have usually been short and the training given can be fairly described as work at high pressure. On general educational grounds we do not believe that improvements so effected are likely to transfer. Further, the processes tested in the laboratory investigations are comparatively simple. They differ in marked fashion from the complex processes involved in Latin, mathematics or science. In short, the conditions of the experiments depart so radically from ordinary class-room conditions, it may well be questioned whether results so obtained can possibly determine even approximately the amount of transfer possible in the case of the school subjects. Even where experiments have been carried out in the schoool room, as by Winch and Sleight, the methods of securing greatest transfer have not been fully utilized. It seems reasonable, therefore, to conclude that even if but slight transfer effect has been found in experimental work, yet judgment against a wide spread of improvement, in general, should be suspended, since the conditions favorable to generalization were absent. At present we have only prophecy, not knowledge. The experimental results to hand are so paltry and limited compared with the mass of facts to be measured in any reliable attempt to establish the extent of transfer in the case of any of the secondary school subjects, that

an extreme view stands discredited. It is credible, to say the least, that the amount of general effect produced by the high-school training in mathematics is greatly in excess of the laboratory figures. But measurement of the actual changes made by mathematics has still to be accomplished.

Psychology has a more positive contribution to make as regards the second point in dispute. There is, to be sure, a variety of opinions as to the ways and means by which transfer is facilitated. Identical elements, development of attention, will, mental imagery, ideals, divesting the essential process of inessential elements, improvement in the technique of learning—all of these have been suggested as causes of transfer and probably all do function to some extent in the spread of improvement. There is, however, a growing consensus of opinion upon the factors operative in transfer, as a result of the progress made by those investigators, who have subjected their numerical and introspective results to careful analysis. Prominent among such studies is that of Ruger. In this article his conclusions can only be briefly summarized. He found four general factors in transfer of training—ideals, attitudes, concepts of method and high level attention. The formation of ideals, such as a general idea of efficiency, was an important element in the spread of improvement from one activity to another, and similarly the attitude adopted facilitated or hindered transfer. Thus a self-conscious attitude restricted progress and checked transfer, while an attitude of self-confidence was very favorable. Again, concepts of method played a most important part. The conscious control of assumptions, the active search for new hypotheses, the effort to distinguish between suggestions and to classify them appropriately, the deliberate testing of hypotheses, and the generalization of these methods themselves together with the realization of the value of such generalizations greatly contributed to transfer. Above all a high level of attention was an indispensable precondition. Of the special factors in transfer the most important were related ideas. Upon these depends to a large extent the possibility of generalization. We have been apt to believe that if we gave individuals a theory, they would be able to apply it appropriately; but the fact is that children have to be as carefully taught to apply theory as to un-

derstand it and unless related ideas are pointed out to them they will often fail to perceive their connection. Ruger found in his experiments with wire puzzles that geometrical concepts played no part in hastening their solution and that the greatest transfer was from similar puzzles. Finally as regards the relation of habit to transfer it seemed that "the mere presence in the case of change of conditions of motor habits appropriate to the new conditions did not necessitate positive transfer. The degree of positive transfer varied directly with the precision of analysis of the similarity of the new case to the old."

These results suggest the following practical injunctions. First, proper attitudes should be cultivated in the pupil. Secondly attention should be focused on the art of learning and on methods of procedure in the solving of problems, so that they should be stimulated to analyze the situation, to formulate hypotheses, to criticize and evaluate each suggestion, to be systematic in selecting and rejecting these and in verifying them. Further, each step should be generalized as a method so that there should be deliberate control of assumptions. Thirdly, attention should be directed to related ideas in order that as many as possible may be recalled or discovered. Lastly, motivation should be secured and attention be kept at a high level. By such means and in such ways the experience gained in mathematics will tend to be generalized and made available in other fields.

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TEACHERS COLLEGE,
NEW YORK CITY.

RECENT CRITICISMS OF MATHEMATICS TEACHING, AND THEIR RESULTS.*

EASTERN NEW YORK STATE.

BY ARTHUR M. CURTIS,

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The following questionnaire was sent to about twenty representative school and college men: city superintendents, college professors, normal and high-school instructors.

I. Who are the leading critics of mathematics teaching?

II. What are their viewpoints?

III. What changes have you noted in the time allotted to mathematics in high-school courses and in college courses?

IV. Have there been appreciable changes in the arrangement of the courses?

V. What changes in length of courses do you suggest?

VI. Is the preparation of your entering classes (in high school or college) improving or declining?

VII. To what do you ascribe this change (in No. VI.)?

N. B. The questions refer to a period of time not more than five to ten years back from the present.

Their answers were as follows:

I. (*a*) The teachers of mathematics themselves.
   (*b*) Teachers of other subjects in the curriculum.
   (*c*) Many individuals mentioned, authors, etc.
   (*d*) "Half-baked" psychologists.
   (*e*) Vocationists.
   (*f*) The public—utilitarian public.

II. (*a*) Dissatisfaction with their own results.
   (*b*) Jealousy of time demanded by mathematics in lesson preparation.

* Reports from different parts of the Middle States and Maryland given at the spring meeting. On account of their similarity it has seemed best to condense them into one article.

(*c*) All forms of "isms" from the Snedden viewpoint to the extreme classicist—the *old* school.

(*d*) The denial of transference, etc.

(*e*) Everything must be immediately useful and have an *earning* value.

(*f*) Quite like (*e*).

III. (*a*) None noteworthy.

(*b*) More time (one and one half years in many schools) for elementary algebra.

(*c*) Making algebra elective (in prospect, not as yet accomplished).

(*d*) More time for analytics.

IV. (*a*) Elementary algebra to be followed immediately by intermediate algebra, rather than separated by two years.

(*b*) Analytical geometry and calculus in one course.

V. (*a*) Same as III. (*d*).

(*b*) Geometry in first-year high school.

(*c*) Shorten elementary algebra to the study of the equation.

(*d*) Place arithmetic in high school course.

(*e*) "Don't take the 'pep' out of the subject."

(*f*) Emphasize principles in the courses and rapidity in process.

VI. Eight answered "Declining,"
Four answered "The same,"
Three answered "Better,"
and a few gave no opinion.
One—"Some send good always and others send poor always."

VII. (*a*) Too little of the joy of achievement.

(*b*) Effect (for "Better") of scholarships now given by the state.

(*c*) Poor ineffective teaching in first-year algebra inducing a dislike for the subject.

(*d*) The looseness in thought expression in other lines contributes to inaccuracy, and is most apparent in arithmetic.

(*e*) Too many subjects and not enough concentration of effort.

(*f*) Outside interests too numerous.
(*g*) Lack of "background."
(*h*) Emphasis on memorizing instead of thinking.
(*i*) I see the facts (of decline), but do not know to what to ascribe them.

New York City.

By Forrest A. De Graff,

Stuyvesant High School.

To secure the information asked for in questions (*b*) and (*c*) of today's program, a questionnaire embodying them was addressed to fifteen high schools and five colleges in New York City, as well as to several representative institutions in nearby municipalities whose industrial problem seemed to invite adjustments of mathematical courses. Although several replies are lacking, an effort has been made through personal conferences and perusal of catalogs to supply the missing information.

From some quarters there are evidences of serious disturbances, while in others we find quite well developed plans for industrialized and commercialized mathematics designed to still public clamors, and at the same time soothe the well-grounded fears of that other group who are content with the traditional.

That the *teaching* of mathematics should show a rational motivation, that the subject should be shown to have a broader adaptation to the affairs of daily life, and that there should be given a greater measure of liberty to the more mature pupil in the matter of choosing certain mathematical subjects for study —these three considerations appear to be quite generally accepted as sound and proper lines, along which reforms of courses may be safely pursued; but hysterical demands for a thorough housecleaning (prefacing, I suppose, a general removal) have met with comparatively little sympathy in this vicinity, and the present prospect fails to show any general trend toward such a realization of the hopes of certain enthusiasts.

The influence of the utilitarians is everywhere observable, but the concept of utility has been dignified, has lost much of its cheapness, and in some of our schools, it has become the central

idea around which strong, well-balanced courses of real mathematics are being constructed. There is also abundant evidence that another well-known exhortation, "Every subject of study should be made to function in the life of the child," is being carried out without sacrificing what the *adult* may require, thereby transferring the emphasis from the *child* to *life*, and that in its broadest sense.

The transferring of several topics of elementary algebra to intermediate by the regents seems to have met with general approval, chiefly because time is thus released for some other required subject or an opportunity is thus given for supervised study and drill. The omission of certain propositions from geometry, determinants from advanced algebra, and the permission to use the text of plane trigonometry during examination have all been accepted more or less gracefully for similar reasons.

In the technical high schools such as Stuyvesant we have shop mathematics as a doorway to algebra largely through the medium of the formula, also the courses in applied mathematics for co-operative students who alternate school and shop, and whose mathematical needs require the study of the whole range of elementary subjects from the practical viewpoint. The commercial high schools are meeting the demand for the utilizing of mathematics by working out courses in commercial algebra, and in making all mathematics more vocational.

In one commercial high school, however, all mathematics, except commercial arithmetic, is made elective, with results that, needless to say, are very unsatisfactory to the mathematics department. The statement is made that the course is not comparable to any other in the city high schools, and the hope is expressed that before long a more extensive one will take its place. Changes of like nature are being discussed at a very large girls' industrial high school. In these two schools only does the course of real mathematics seem to be languishing for the want of stronger administrative support.

Only one out of twenty high schools reporting indicated a change in the amount of time allotted, and in this case there is a slight reduction. The same school, together with another of the same type (commercial), shows a decrease in the number of students electing mathematics.

In Columbia, New York University, Fordham, Adelphi and Barnard we find no change in the allotment of time given to mathematics; and there is an appreciable increase in the relative number of students in Columbia, Barnard, and Fordham. Adelphi remains unchanged in this respect, while New York University shows a slight relative decrease. Adelphi will give less solid geometry and more trigonometry, also less theory of numbers; mathematics has been optional, however, since the beginning of the college. No change in the arrangement of courses and no reconstruction are reported in Columbia, Barnard, and New York University.

Middle and Western New York State.

By Floyd F. Decker,

Syracuse University.

Through this district there is little criticism of mathematics teaching. The requirements in school and college have not been changed to any appreciable extent. The question as to the relative number of pupils taking high-school mathematics seemed hard to answer definitely. The following table, however, gives the number of pupils per thousand of enrollment who presented answer papers to the regents in the years shown.

	1904–6.	1905–7.	1906–8.	1907–9.	1908–10.	1909–12.	1910–13.	1912–13.
All mathematics	328	380	439	494	510	513	518	528
First algebra	178	199	224	246	250	250	254	265
Plane geometry	112	128	142	157	159	162	166	168
Second algebra	10.5	20.2	33.6	48.8	56.2	58.5	58.9	58.4
Third algebra			8.5	9.0	12.5	9.3	9.0	9.1
Solid geometry	19.1	19.9	21.4	22.2	22.3	21.6	20.7	20.6
Average of plane and spherical trigonometry	6.5	7.8	9.2	9.8	11.1	9.9	9.3	6.9

In the college course there is a tendency to combine analytic geometry and calculus, and courses giving a general view of mathematics are being added.

Western New York.

By Wilfred H. Sherk,

Lafayette High School, Buffalo.

In gathering the material for this report of the results of the recent criticisms of mathematics teaching in western New York, a questionnaire was sent to one hundred and twenty high schools and to ten colleges. Three of the colleges were not in the section covered by the report. The statistics compiled from the replies do not necessarily indicate the ***trend*** of mathematics teaching in this section, for the statistical method gives just as much weight to the opinion of the least progressive and the least competent teacher as to that of the most progressive and the most competent. However, these statistics do furnish a general notion of the teaching in this territory as it actually is, and this, I understand, is the object of the survey.

Vitalizing Mathematics.

In the first place all of the high schools from which replies were received are trying in one way or another to vitalize mathematics. They are trying to discover and to use natural points of contact of mathematics with the other interests of the pupils. It does not appear that teachers are consciously emphasizing the applications of mathematics, as opposed to its intrinsic value or its potential utility. They have simply found as a matter of experience that applications which are real and which connect mathematics with the rest of the pupil's life are interesting and furnish an excellent motive for studying the subject.

Laboratory Method.

With reference to the laboratory method it may be said that in arithmetic many of our grammar schools are using it with success. Children weigh, measure, buy and sell, make change, write checks, make out bills and payrolls, and so forth. If under the caption "laboratory method" we include study under supervision, special work undertaken to assist backward or brilliant pupils, and all that work on the part of the pupil which

is done under the immediate direction of the teacher, then the laboratory method is used very extensively in western New York. The number of schools that are encouraging study under supervision is noteworthy.

Parallel Courses; Correlation; Fusion or General Mathematics.

Most of the schools in this section still teach arithmetic, algebra, geometry and trigonometry in the traditional sequence. Correlation of the mathematical subjects among themselves is on the increase, but no school reported that it was attempting to correlate mathematics with any other subject except, possibly, drawing. This tendency toward correlation was especially noticeable in the larger schools. Algebra is correlated with arithmetic by introducing the algebraic formula and the linear equation; geometry with arithmetic through mensuration; geometry with algebra by the introduction of simple geometric problems capable of algebraic solution; trigonometry with geometry by the introduction of trigonometric ratios in connection with similar triangles; surveying with trigonometry through the solution of triangles.

Rochester in its junior high school and Buffalo in the Hutchinson high school are running algebra and geometry in parallel courses with much correlation between the two. There is no secondary school in western New York that I know of in which fusion or general mathematics is being taught. It is worth noting, however, that parallel courses between which there is a great deal of correlation may easily become fused.

The question of the wisdom of requiring algebra and geometry of all high-school pupils has not been raised in western New York, and the number of pupils electing mathematics is on the whole increasing.

In the colleges Dr. Gale, of the University of Rochester, is trying to unify the work of the freshman year around the function idea. Other college men in this section, while not actively participating in the experiment, believe that it is being conducted along right lines, and are awaiting the results with a great deal of interest.

MARYLAND.

BY EUGENE RANDOLPH SMITH,

The Park School, Baltimore.

There has been little change in either school or college mathematics in Maryland. Some unimportant criticisms of the subject have been made and answered, but without effect on the general procedure. Individual schools are using the more modern methods in arithmetic and are doing progressive work in the high-school subjects, but the state as a whole has not been greatly influenced by recent developments.

The Johns Hopkins University still requires plane trigonometry in addition to the usual subjects for entrance to the academic course, and requires all freshmen to take two of the three subjects, analytics, physics and Latin. The fusion of subjects in the freshman and sophomore years has been quite thoroughly developed.

Goucher College has just announced that pupils may enter on a comprehensive examination plan, which makes it possible, but not easy, to enter without mathematics.

NEW JERSEY.

BY HARRISON E. WEBB,

Central High School, Newark.

Mr. Webb had not prepared any questionnaire but had made a good many informal inquiries. He felt that the chief critics of mathematics teaching in high school were those teachers or employers to whom students leaving high school were later to be responsible. Many of their criticisms are really directed against conditions which are the result of the present standards of high-school instruction and administration generally, and these criticisms cannot be met primarily by any changes in curriculum or method of mathematics teaching. Others demand our earnest consideration, among them the chief one being that students are unable to apply what mathematics they have either

to practical work or to advanced study. The general tendency in New Jersey is to lay greater emphasis upon applications of mathematics, in order to meet this criticism, and also on account of the enormous increase in high-school attendance in the larger cities of New Jersey, most of which will never find its way into college.

This emphasis upon the application of mathematics is shown in three ways:

(*a*) The introduction of arithmetic into the first year of high school. This is based on the idea that there are many important topics in arithmetic which are not suited to the relative immaturity of grammar-school pupils. This work has hitherto been distributed among various other subjects in mathematics or science, but is now often given under its own caption.

(*b*) Applied problems are introduced freely in algebra and geometry.

(*c*) Courses in plane surveying, strength of materials, and business algebra, almost the entire content of which is practical, have been added to the mathematical curriculum.

MATHEMATICS AND PSYCHOLOGY.

By C. C. Grove.

(*Continued from page 10, Vol. IX.*)

Part II.

The Mathematics of Psychologists and the Validity of their Uses of its Forms and Processes.

The "measurements" herein spoken of as *indirect* are so called because they are physical measurements upon the physical causes or effects of mental activity. "They derive all their significance from the correlated psychical processes," says Brown, l. c., p. 11. They belong to the period of investigation that may be referred to as that of "mental tests." Their name is legion. This came about owing to the early misconception that any psychological tests would serve the purpose of "mental tests," and also to the fact that many without a clear or an adequate conception of just what they were testing went on applying their inventive genius to devising test after test as if to outdo all other workers in the cleverness of their tests. Prominent workers even presupposed that this or that was the essential element and measure of a particular mental process, without testing whether there was any correlation even between them. Reference is made to this in the excellent monograph "The Psychological Methods of Testing Intelligence," by William Stern, translated by G. M. Whipple. Warwick & York, Inc., Baltimore, 1914, in the words (p. 24): Very seldom was any actual preliminary investigation made to see whether this particular test was really superior to so and so many others by virtue of the precision, constancy and significance of the particular values that it afforded.

The day has not yet passed of devising single tests or series of tests without apparent concern as to what is really being tested or as to the scientific superiority of the test, for only last

January across there at the University of Pennsylvania we heard Section L of the A. A. A. S. enthusiastically introduced to a test to have been published last September, whose chief charm seemed to rest in the fact that "the children fairly ate it up."

William Stern laid a scientifically firm foundation by first defining clearly what he was testing, viz., Intelligence, l. c., page 3:

Intelligence is a general capacity of an individual consciously to adjust his thinking to new requirements: it is general mental adaptability to new problems and conditions of life.

By this definition he is enabled to distinguish intelligence from *memory,* which has to do not with the *new* but with the past, from *genius,* which is characterized not by *adaptability* but by creation, and finally from *talent,* which is not *general* but specific *capacity.*

The methods of testing intelligence are divided in the monograph into four types—*Single Tests, Series of Tests,* the *Method of Age Gradation,* and the *Method of Rank Correlation.*

On pages 13–18, he states and discusses four main classes of *single tests,* both indirect and direct, and expresses his conviction of the inadequacy of the single test by "laying emphasis upon a general critical position: *no single test, no matter how good it may be, should ever be made the instrument for testing the intelligence of an individual.*"

All this, the definition, discussion and conclusions, appeals very strongly to the present writer because of its clarity and truly scientific spirit. Several quotations are now inserted because they bring corroboration from an eminent psychologist to the writer's own views to follow.

P. 8: The consequent demand for the creation of normal test-series for each year of childhood was met, as a matter of fact, not from the side of psychiatry, but from that of psychology. Alfred Binet, with the co-operation of the physician, Simon, has created such a graded series of tests; and although the system as it now stands may be far from final, its fundamental conception will retain its permanent value and will doubtless lead us ultimately to a completely satisfactory solution.

P. 26: This demand for a system of tests presents such an

exceedingly difficult scientific problem that it is perfectly evident that alienists and educators can not solve it as a side issue of their professional work, but that psychology itself will have to undertake the task. It is interesting in this connection to note how psychology attacked the problem along two very different lines: The *method of age gradation of Binet and Simon permits of a rough* gradation of intelligence for the whole range of development of the child; it *is for use in a comparable manner* with children of different ages, of different nationality and cultural level, with normal and with feeble-minded children of all grades. The *method of rank correlation* is limited thus far to a comparison of the members of a small homogeneous group, but renders it possible to test the gradation of intelligence within this group with a precision that the Binet method cannot approximate.

Pp. 11, 12: But just this anticipated extension of the practical applicability of intelligence tests necessitates several words of warning.

(*a*) We are still in the midst of our *preliminary work* on method. The methods that now prevail—and this is true also of the Binet-Simon system—are not yet to be regarded as diagnostic canons that admit of official prescription. The law passed in New Jersey that directs the use of intelligence tests with all pupils suspected of backwardness seems on this account very premature. So, too, it will be long, very long, before we realize the optimistic hope that Spearman attaches to the correlation method of testing intelligence, when he says: "Indeed, it seems possible to forsee the day when there will be an annual official determination of the 'intellectual index' of every child in the empire."

(*b*) It must be understood that tests of intelligence are not easy to conduct. Their administration demands extended practice, psychological training, and a critical mind. Thus, for instance, the average teacher, whose work has been with the wholly different methods of pedagogical questioning and examining, is very apt to apply psychological tests in those forms in which their value would be positively illusory. If, accordingly, the use of tests for practical purposes shall attain any very large currency, the training of a specially psychologically drilled per-

sonnel will become a necessity. School psychologists would then take their place side by side with the school physician.

(*c*) Psychological tests *must not be overestimated,* as if they were complete and automatically operative measures of mind. At most they are the psychographic minimum that gives us a first orientation concerning individuals about whom nothing else is known, and they are of service to complement and to render comparable and objectively gradable other observations —psychological, pedagogical, medical—not to replace these. (See *Jour. of Educ. Psychol.,* 2: 1911, 121 and 191 by J. E. W. Wallin; also "Manual of Mental and Physical Tests," by G. M. Whipple.)

P. 33: The individual tests are of unequal value.

P. 34: The *technique* of the Binet-Simon method is by no means so easy as the simplicity of the material used would lead one at first to suppose. It is to be recommended that, so far as is in any way feasible, the examiner should always do his work with the aid of an assistant to keep the record, so as to avoid the undesirable division of attention between testing and recording. Both *these experimenters must have gained a high degree of practice* and be well used to one another *before they proceed to actual testing.* The examiner must have an almost mechanical exactness and uniformity in the formulation of the continually recurring questions, in the modulation of his voice, etc., yet he must be prepared for the many individual variations that appear in consequence of different reactions of the subjects, and must have definite measures in readiness for use in these junctures. Never must he permit it to be seen that some answers are more, others less satisfactory to him: rather must he maintain an attitude of uniform and quiet friendliness. The recorder should not confine himself to the mere recording of plus and minus signs to show the net outcome of each test, but should also note down as fully as possible what the subject says and also such features of his behavior and attitude towards the tests as are worth noting. This is necessary both because it is often impossible to decide whether to credit a test "plus" or "minus" until later on, after quiet consideration (and the material must be available for that) and also because it should make possible a qualitative analysis of the examinee.

P. 40: The value of observing the child during the testing must not be underestimated, for in many of the tests there are ways of setting about the task that may be of great interest (and for medical and pedagogical judgment of the case, too), though these things would not be evident from the mere plus or minus set down for the outcome of the tests.

P. 49: The *international accordance* in the judgment as to special ease or special difficulty of certain test levels, despite the differences in race and language, despite the divergences in school organization and in methods of instruction, is, in my opinion, the best vindication of the *principle* of the tests that one could imagine, because this agreement demonstrates that *the tests do actually reach and discover the general developmental conditions of intelligence* (so far as these are operative in public school children of the present cultural epoch), and not mere fragments of knowledge and attainments acquired by chance.

P. 58: Tests of intelligence have already taught us that the relations between intelligence and school ability are by no means so strict and uniform as most persons had thought.

P. 63: Complete agreement between school ability and intellectual ability is not to be expected at all nor even to be desired, *because performance in the school depends not only upon intelligence, but also upon certain other and quite different factors.* . . . But besides this there are concerned factors that have nothing at all to do with intellect, but belong to the domain of will, in the widest sense of that term: I mean the degree and duration of attention, industry and conscientiousness, sense of duty and capacity to fit into the social group.

P. 64: Intelligence is never more than a partial factor in school activity: and this demonstration may serve to refute that one-sided intellectualism that notes and values in pupils only their intellectual ability. Not that intellectual endowment is not still to be regarded as a factor of chief importance: in truth when by tests of intelligence and other psychological devices we shall have obtained a more exact knowledge of it, there will be much of profit for the schools and many mistakes and wrong courses of procedure can be prevented, and this so much the more as we get clear ideas of the range and limits of its meaning

and importance. If, for instance, a given pupil shows only a moderate success in the tests of intelligence but does distinctly good work at school, and if there is no chance that a special talent might have exerted a decided influence (which could easily be recognized if existent), then there is a probability approximating to certainty that this pupil's strength is to be sought primarily in qualities of character and will.

P. 9: . . . we must guard against the danger . . . of supposing that we have grasped the individuality of a pupil in its totality when we have tested his intelligence. The fact that intelligence can be more easily treated quantitatively than can other individual capacities must not lead us to overestimate its import. Nevertheless, the fact that we can deal with intelligence by itself does serve to disclose the structure of the individuality. We can determine whether a performance of greater or lesser degree depends on talent or on intelligence; we can investigate what degree of correspondence exists between the experimental results and the teacher's judgments of the intelligence of pupils; we can delimit the extent to which general school efficiency is dependent on intelligence itself on the one hand and on non-intellectual factors on the other hand—a delimitation that forms one of the chief merits of the psychological methods.

The present writer's judgment on matters psychological is not to be considered. We shall let the psychologists speak for us here. Yet, careful study extending over a considerable time has brought the conviction that two notable, important and fruitful workers, my colleague Professor E. L. Thorndike and C. Spearman, of the University of London, both, but especially the former, through their enthusiasm, exuberance, genius, and a rather natural carelessness of details and disregard of foundations and fundamental assumptions, have perpetrated on workers in other fields ideas, opinions, surmises, as scientifically established facts, in a way that a more scientific and careful worker could never have done. Only one of rare genius can touch with acceptable suggestions so many different fields of scholarly endeavor as has Professor Thorndike. Yet this very element of genius may have robbed his writings and public addresses of that desirable scientific flavor, and made it possible

for psychologists of such note as Titchener, *l. c.*, p. 99, E. B. Holt, *The Jour. of Abnormal Psychology*, Jan. '16, pp. 366–7, and C. H. Judd, "Psychology of High School Subjects," to furnish us with such reviews, lamenting a general carelessness of exposition, obscurity, and even incoherence and equivocal expression. The writer would call attention to such implied and insinuated meanings as illustrated by the title of a paper presented at the Cincinnati meeting of the N. E. A., "Can algebra and geometry be reorganized so as to justify their retention for high school pupils not likely to enter technical schools?" and also by several sentences in the address on "Educational Diagnosis," *Science*, Jan. 24, 1913, pp. 133 sq.

Be it clearly understood that all this is not to be taken as an attack but as an analysis of the causes of the rather remarkable influence of the work of Professor Thorndike and those who have followed him. We all, the writer takes it, are working towards the same end, and with the same degree of sincerity, and therefore find neither pleasure nor interest in attempts at weakening another's position. We are all searchers for truth, regardless of what that truth reveals, and without preconceived ideas that we wish to establish. Relative to the statistician, let me say, that no scientific worker needs more to have the qualities of the ideal scientist, sincerity, honesty, open-mindedness, and zeal for truth. This is true because there is for him far greater temptation to try to corroborate preconceptions, to establish pet hypotheses.

A fellow-student with me at Johns Hopkins University was several years ago asked to call on an official of a large corporation that needed an expert statistician. After a bit of conversation the official said, as if in great surprise: "Did you not say that you are a statistician? But you are a scholarly gentleman." This official's conception of a statistician is alas all too common and it is strengthened when men who are scholarly and stand high in some line of research use the methods of statistics blindly or without understanding fully the fundamental principles and assumptions. The work of statistician should be raised from its low estate, and might be if all statisticians had to be certified as are accountants.

Reverting to the series of quotations, we shall consider the

third, from pp. 11, 12: (*a*) It seems quite clear that those under discussion do not at least keep in mind the fact that the work is still in the preliminary stage, or rather did not when that was more the case. Much has had all the marks of being regarded as a diagnostic canon, and its influence has been all the same as if it was. (*b*) On this point the writer would refer to the Report of the Arithmetic Committee in this journal of March, 1915, pp. 125–6. The greatest amount of *joint* discussion on that report was had during the hour that three of the committee searched for a bite of lunch along Broadway just before the report was made. This furnished poor opportunity for me to secure more than modifications, so that "suggestions and recommendations," Nos. 1, 2 and 6 remained, under my protest for the reasons expressed in this "warning (*b*)," then unknown. The writer accordingly endorses most heartily the quotations from Stern, pp. 11, 12, 34, 40, and deplores the time worse than wasted by having every young school teacher conducting tests on the strength of a few weeks spent in a course in educational psychology or without any special training whatsoever. (*c*) Psychological tests have been overestimated, and have sometimes replaced and interfered with the teacher's getting the spirit of her subject and of transmitting it to her class.

Further on p. 34, we note that long before psychological tests were made, Pope observed what is a great but much neglected factor in the conduct of all such tests:

> The difference is as great between
> The optics seeing as the objects seen.
> All manners take a tincture from our own,
> Or come discolored through our passions shown.

This interaction between the tester and the tested as well as the fact that every one realizes more or less (and in an indeterminate degree) that he is being tested, make any pretense at precision in these so-called measurements futile. And if there is no claim to precision, why strain over the instruments of precision, the various coefficients, etc.? A line may be run and surveying done in the wilds of the Adirondacks with a Jacob's staff, but along Wall street the finest transit is required. The legitimate use of the instruments of precision is pointed out by Stern in quotations from pp. 8 and 26, the first two herein given.

A graduate student recently, in defending "mental measurements," said that they are now in the stage analogous to that of measuring the intensity of light when a candle was used as the unit. He did not recognize that the "international candle" is simply a far more refined unit, just as the standard meter is more refined than the length of a man's foot. Continuing his figure, we may add that in measuring light the matter is very much complicated when quality or color enters in. In psychological "measurements" the quality factor is preponderant but largely ignored. Spearman's plan of taking very few observations and making hypothetical general corrections on stated assumptions is a way that makes no appeal to a man who has the scientific method in his blood. To such jugglery, the latter quietly shakes his head and *sotto voce* says *"Ignoramus."*

The essential of measurement is nicely stated in Boole's "Finite Differences," pp. 106–7, thus: "In describing a magnitude there must be a standard magnitude of the same sort 'recognized alike by ourselves and those whom we address.' Of vague and indefinite expressions, 'very tall,' etc., we are never certain that those with whom we are conversing 'attach exactly the same idea to the expression that we do.' This vagueness is of little consequence in common life, because in most cases it is impossible to make an accurate estimate." This indispensable quality does not appear at all clearly in "mental measurements" nor yet in the formation of so-called *scales,* of which we shall speak briefly later. We can test a bar of steel in the laboratory, but we cannot measure the things of the spirit of man. One has said, "When I am weak, then am I strong." The testing goes on and the results seem regular, until the limit seems near, when lo, something else comes in. The person seems to get "second wind"; in weakness, comes a new flood of strength. This phenomenon is doubtless known to you all in personal experience. It has been put into exquisite verse by our "children's poet":

> As torrents in summer,
> Half-dried in their channels,
> Suddenly rise, though the
> Sky is still cloudless,
> For rain has been falling
> Far off at their fountains;

So hearts that are fainting
Grow full to o'erflowing,
And they that behold it
Marvel, and know not
That God at their fountains
Far off has been raining!

Nevertheless, *tests* have proven their use, some sorts without calculations of precision and others with the assistance of such forms of computation, but *all only when properly administered by scientifically trained experts.* The remaining quotations from Stern, pp. 49, 63, 64, and 9, were included to testify to this. Prof. S. A. Courtis has written, in *The Elementary School* Teacher, April, 1914: "In the writer's thinking, the functions of standard educational tests are four: (*a*) to secure information that will enable school authorities to formulate in objective terms the ends to be attained in any educational processes; (*b*) to measure the efficiency of methods designed to produce the desired results; (*c*) to determine the factors and laws which condition learning and teaching; (*d*) to furnish data that will enable comparisons of school with school, and teacher with teacher for purposes of supervisory control to be made upon *scientific, impersonal, objective* bases." The extent to which these desirable ends are being attained is not very clear. In his annual report, pp. 120–122, over date of Oct. 15, 1914, President Pritchett, writing on educational surveys, says: "There is, in fact, some danger that the faith in statistics may overshadow the main purpose of the school—the education of the pupils. The tendency to neglect the product by all-absorbing attention to the process is too marked at present to be overlooked." All three pages should be read to get his view of the state of these matters of supervision and administration.

We are brought thus naturally to give the promised examples of classes of errors and misconceptions found in the literature, for "Educational Administration—Quantitative Studies," Strayer and Thorndike, furnishes a good illustration of the way in which statistical data are used. On p. 100, a study of the social and economic status of teachers is begun. From it we quote, "Who are the teachers of our children? . . . What kind of teachers do we get for the money we pay? Is there any relation between the

amount of salary a teacher receives and the amount of training secured by him? From what social group do teachers come? These and many other similar questions must be answered by anyone who would attempt to judge of the efficiency of our public systems of education. In the investigation by Professor L. D. Coffman, entitled 'The Social Composition of the Teaching Population,' we have the answer to our questions.

"Dr. Coffman's research is based upon the answers received to a questionnaire which was answered by 5,215 teachers selected at random in seventeen states. Most of the answers were secured from teachers who were in attendance upon their annual institutes. The purpose of the questionnaire was explained and replies were received from *all* of those present. . . . The tables are in the main self-explanatory.

"When we ask, Who are the teachers of our children, we must inquire concerning the families from which teachers come. The social status and the income of the parents of teachers limits the social inheritance which these teachers transmit to children. The following tables giving the occupations of parents, their income, and the number of brothers and sisters present a clear picture of the social and economic groups represented by the families from which teachers are recruited."

The affirmation that "we have the answer," and the statements, "selected at random" and "replies . . . from *all* . . . present," inserted to strengthen belief in the former, might be quite amusing if they were not intended to be serious, especially when we read (between the lines, in part) that most of the replies were from teachers assembled in their *county* institutes, from their rural schools in Middle Atlantic and Middle Western states. Rather "a clear picture" of this is obtained and corroborated when we see that 687 men out of 985 who stated the occupation of parent, and 1,409 out of 3,153 women who stated occupation, answered "farmers." What else would you expect? and what does it all mean? And then the assumption as to social status and income—well, what does every thoughtful student reader think as he studies the pages? We are sure Professor Thorndike would not for a moment think of claiming that the income of his father, a Methodist minister, and his own present annual income are in any way proportional to the

real worth of the two men to their day and generation, or even of their social status in their respective communities. All real men do not know their worth and much less lay claim to it. A keen observation is portrayed in the parable in the asking, "When saw we thee sick, or in prison, and came unto thee?" You can't measure worth that way.

Examples might be multiplied to show that unwarranted, overdrawn or trivial conclusions are drawn, but we must close with but two more. Table 38, p. 131, relates to "Relative frequencies of different salaries in public and private secondary schools in the same localities. Percentages estimated from nineteen cities." Now a moment's reflection tells us that many of the country's best private secondary schools, paying the highest salaries, are not located in cities, and that the best schools, both public and private, are more likely not to be "in the same localities" for defective schools of one type fosters better schools of the other type. Of what real worth in drawing conclusions can such a table be? Surely we cannot speak of "accurate measurements" as is done elsewhere in the book. On p. 239 amongst other conclusions we find, "a large amount of time spent on arithmetic is no guarantee of a high degree of efficiency." Why must months of valuable time for pupils and teachers be spent to reach this profound conclusion, manifest to any thoughtful teacher or superintendent? It seems like a fifty-page sociological study that concluded that the Lower East Side is most densely populated and mostly by Jews, or like the sage conclusion that training in addition does not increase reasoning ability. Who with any power of introspection and observation ever claimed the latter untrue, and that without accusing himself of (p. 132) "the intellectual crime of giving mere opinions where indolence is the only excuse for failing to verify them." Surely nobody can charge the statistic users of indolence, yet the type of conclusions just cited recalls the excellent line of Horace ("Ars Poetica," l. 139) descriptive of some literary productions:

Parturient montes, nascetur ridiculus mus.

Turning next to "Theory of Mental and Social Measurements," E. L. Thorndike, The Science Press, 1904, we shall

mention only the first two pages, saying that their statements and spirit are considered not expressive of their writer's present thought on the subject, that as they appear several statements are contrary to fact and their spirit is such as to misrepresent both mathematics and statistics to the reader. Professor Titchener (l. c., p. 99) has ably reviewed this work, and closes with the remark: "In any event, however, a mathematics for the psychologist has still to be written." Several years of teaching the subject has convinced the present writer that the sort of text most needed to fulfill the demand is one that he has been inspired, in part by the above challenge, to conceive and begin to write. In brief outline, it is to present a rigorous treatment beginning with the theory of the quadratic equation, leading to such topics in the general theory of equations as are directly useful in statistical work or in a rigorous development of other topics that are necessary; continuing the algebra through the topics that lead to a careful study of mathematical probability, it will treat that subject and its train of related subjects, interspersing analytic geometry and the calculus before it is possible to complete that train. The second part is to treat as carefully the various statistical measures, constantly keeping clearly in view the fundamental assumptions made in their development. The book is to contain an exhaustive treatment of correlation that should clear the atmosphere at a point where blue sky has long been hoped for. Read the recent article by Prof. C. N. Moore in *School and Society*, Sept. 11, 1915. See also *Science*, Apr. 24, 1914, p. 609.

Illustrating further the kind of mathematical provender provided for the students of psychology, we cite "Experimental Psychology and Pedagogy," R. Schulze, translated by Rudolf Pintner, The Macmillan Co., 1912. Chapter I., on the Mathematical Treatment of Results, starts (p. 17) with the results of eighty measurements of the length of a certain rod. They are absolutely symmetrical, the frequencies being 1, 3, 8, 17, 22, 17, 8, 3, 1, the range being 0.4 mm. on 100 mm. The author must have practiced some time to have an actual set (so stated) of eighty observations come out so beautifully. Then on p. 19 follows: "Since in our measurements the accidental errors are symmetrically grouped, the arithmetical mean of all the single

measurements will give us the real length of the measured object." Would we might ever know, and at least so easily, the *real* measure of any quantity! Further, in his case, the mean, median and mode coincide so there is no chance to state any comparative values of the three averages, as has been done by another but without proper understanding. From pp. 20, 21, we quote, "Therefore it is always best to calculate the average error," and "it has been established that the probable error always amounts to 4/5 (or, more precisely .8453) of the average error." "Always"—under what assumptions? Of these there seems to be utter disregard or ignorance. Finally on pp. 23 and 24 the crass superficiality and astonishing naïvete of the whole treatment appears in the paragraph:

"The formulæ that we have so far given are accurate enough when more than ten measurements are taken. Other formulæ have been found to be better for a smaller number of measurements. We give below the simple formulæ for reckoning the error that we have just discussed, and append thereto the more accurate formulæ (Nos. 1_a, 2_a, 3_a).

"THE SIMPLE FORMULÆ.

"1. Average error:

$$E_m = \frac{\Sigma\Delta}{n}.$$

"2. Probable error of the separate observations:

$$PE = \pm .8453\frac{\Sigma\Delta}{n}.$$

"3. Probable error of the arithmetical mean:

$$PE_m = \frac{PE}{\sqrt{n}}.$$

"MORE ACCURATE FORMULÆ.

"1_a. Average error:

$$E_m = \sqrt{\frac{\Sigma\Delta^2}{n-1}}.$$

"2_a. Probable error of the separate observations:

$$PE = \pm .6745 \sqrt{\frac{\Sigma\Delta^2}{n-1}}.$$

"3_a. Probable error of the arithmetical mean:

$$PE_m = \frac{PE}{\sqrt{n}}.$$

"Note the fact that even with eighty measurements the accuracy of the simple formulæ is very great." The numerical values from the test are given in each case.

What rhyme or reason there is in dubbing the latter formumulæ "more accurate" is past our finding out. Their real meaning is known to all readers.

The meaningless indefiniteness of the following sentence will be apparent to all (p. 341): "If we define our problem clearly and carefully enough, if we do not make it too wide, the necessity of using that formula can be avoided." The carelessness of the author, or translator and others is shown on p. 347: "The only experiments on these lines known to the author are those carried out by Spearman in AMERICA."

Mention should be made of "Empirical Studies in the Theory of Measurement," E. L. Thorndike, The Science Press, April, 1907, mainly in the way of appreciation. Attention is called (p. 26) to the fact that the assumptions made in the formulation of the Pearson coefficient of correlation "never are except by chance more than approximately" fulfilled, "and in the majority of the cases in which students of the mental and social sciences need to measure relationships, they are far from true." As was stated above, we feel that wherein it is known that data do not fulfill the supposed conditions, it is vain to strive to uphold any particular measure. We should all realize the particular properties of each average and measure, and endeavor to present to our readers the maximum not the minimum of information as to the material under consideration. A timely book in this respect as to averages is the translation by Prof. W. M. Persons of "Statistical Averages" by Dr. Franz Žižek, Henry Holt and Co., 1913. On pp. 5, 6, Prof. Thorndike in

comparing the average and the mean square deviation does so through the *particular* series of samplings he had been using, twice stating the fact. General not particular considerations furnish the basis of final judgment however.

The work of Dr. Spearman could profitably be considered far more than space and time permit. Brown, l. c., p. 89, in a footnote says: "It is perhaps hardly necessary to point out that the detailed consideration of Dr. Spearman's work given here and elsewhere in the present book is, and is meant to be, a tribute to the outstanding importance of that work. No one can appreciate more vividly than does the writer the valuable suggestiveness of all Dr. Spearman's writings, above all in respect to the special precautions therein emphasized as requisite in measuring *psychical* (as distinct from anatomical, etc.) correlation." He therein strikes the key-word, suggestiveness, which earlier led us to couple Professor Thorndike and Dr. Spearman. It seems to us however that the latter's work is more like Herbart's "constructions" to which we referred on p. 4, Vol. IX., No. 1, of this magazine. Any reader desiring to have a careful evaluation of Spearman's work should read Brown, pp. 54–133. In the Proceedings of the 6th International Congress of Psychology, Geneva, 1909, Brown presented a paper in which he attacked Spearman in part as follows:

"Thus the fallacy involved in Spearman's correlation formula is the assumption that variability of individual performance may be classed with errors of observation for the purposes of correlation—an assumption which does not conform with the facts. In view of the difficulties attaching to the correlation formula it is perhaps wiser to leave our original coefficients in their crude form, at least until a method capable of correcting for variability correlation has been devised. The remedy at present would seem to be to increase the number of original measurements of each mental capacity. . . . Much of the work done on correlation by psychologists in the past has been directed towards the determination of the closeness of relationship between elementary psychical capacities, the investigators using the variability of individual measurements as a stepping stone to the discovery of relations which they believed to be relatively constant. These relations, however, may themselves be

found to be very variable. Correlation coefficients may—nay, do—vary from type to type, and these 'typical' coefficients with their relations to one another are likely to prove of greater importance in the science of psychology than those calculated from a mixture of many types. A great deal of work has yet to be done in the field of mental variation, in which correlation will be of continual service, before variations can be at all safely used as a guide to the constant causes or relationships underlying them. On the assumption that a correlation coefficient is a correct measure of the degree of the identity of causation of the two abilities correlated, the question of the numerical relations to one another of all the coefficients that may be calculated between all the possible pairs of abilities correlated is of considerable interest."

Spearman's reply may be found in *The (British) Journal of Psychology,* Vol. III., Part 3, Oct., 1910. His first two paragraphs call attention to "accidental" errors and claim their "size can be measured by the size of the discrepancies between successive measurements of the same thing." Either he does not give the technical meaning to "accidental" errors, or his statement does not agree with the findings in the physical realm. The decision seems to us of no great consequence for, applied to coarse, rough material as is the case in his field, the formula has no *clear* meaning at least. The contention impresses us as analogous to one between two men who, having *stepped* off twenty paces and with a cord staked off a circular plot, are now discussing whether 22/7 or 3.14159 should be used in computing the area. However, Spearman has rendered valuable service in calling to the attention of psychologists the need of caution and vigilance towards detecting hidden factors that effect the values of correlation coefficients. That same caution has likely made workers avoid to a greater extent the wrong idea that adjustment of observations by the method of least squares will give quality and tone to poor, indifferently made observations.

Finally, we bring to your attention a paper in *School and Society,* July 31, 1915, written under Prof. Hollingworth on "An Experimental Study of Self-Analysis, Estimates of Associates, and the Results of Tests." This particular paper is chosen as it is recent, illustrates a number of points, and keeps

the matter in the family. At the end of the second column on p. 172 begins a paragraph that shows that greater care in expression that characterizes more and more some writers at least: "The results show only what happened in this case, and only to that degree do they suggest what we may expect to be generally true. Many similar studies must be made, under all sorts of conditions and by a variety of methods, before we shall have the final answers to our questions."

The method is stated of selecting 25 juniors in college, who at intervals of two to four weeks were given slips of paper bearing the names of all the group of mutually acquainted students, and requested to arrange them in an order of merit as to the possession of the traits, neatness, intelligence, humor, conceit, beauty, vulgarity, snobbishness, refinement and socialibity. A similar group of seniors judged themselves and one another in the same way. The groups were also given well-known psychological tests such as Completion, Association, Opposites, Mixed Relations.

In the second column, p. 173, we read "her median or true position" and in *The Jour. of Phil., Psy. and Scientific Methods*, Oct. 8, 1914, coming also from Barnard College (p. 578), "the average position for each item was calculated. The order of these average positions was considered the objective order," then "this objective or correct order." At the risk of being considered priggish, we call attention to these words "true" and "correct," because they illustrate a fundamental consideration, a considerable factor, in the influence that the psychological work has had. The insinuation of the false was the method of the Serpent of old; and yet, in more serious vein, we must confess that such insinuation, antipodal to the scientific method and spirit, always makes our spirit feel that liberties had been taken with it.

The above is closely followed by this:

"That is to say, the individual's error in judging herself is somewhat greater than the average error of her friends in their judgments of her. The individual does not judge herself *so* accurately as she is judged by her friends."

Surely faith in statistics has run away with both good judgment and common sense!

(P. 177.) "In the long run it is true that she who knows herself best is the best judge of others, or at least tends to be a good judge. The degree to which this is true, however, varies with the trait in question. With the 'undesirable' traits of snobbishness, conceit and vulgarity the coefficients are so low as to be quite unreliable and perhaps due only to chance. The same is true of neatness. But in the cases of refinement, humor, beauty, sociability and intelligence the coefficients are fairly high."

The twenty-five must arrange *all* for specified qualities—some desirable, some undesirable—when presumably some are deemed as not possessing certain of those qualities. How can I classify 24 colleagues say, as to their degree of guilt in the matter of beating their wives, when to the best of my knowledge none of them have ever thought of resorting to that as a form of recreation or of exercise of authority? The article as a whole took us back to a worn, dusty copy of *The Gettysburg Mercury*, our college monthly, wherein we had published some junior thoughts on "Import of Interpretations" of nature and of man's acts, noting that "There *is* a difference in the interpretations and that too dependent upon the former experiences of the observer. We readily recognize a Selective Absorption in mind as well as in matter." That "the subjective element is as manifest in the interpretations of man's acts as of nature," also that "some people see honor, integrity and uprightness wherever there is the slightest semblance of it. Why? Because it is what they would expect to find in their own acts under similar circumstances," and that "your true feelings, being known, enkindle the same feelings towards you," and finally "if you find yourself constantly adjudging men as honest and trustworthy or as false and hypocritical, you may be well assured that these are characteristic of your own life and that too to a greater degree than you may be ready to admit. Paul says, 'Wherein thou judgest another, thou condemnest thyself.' Our judgments have ever a tincture of our own character." David Hume in 1817 said: "There is a universal tendency among mankind to conceive all beings like themselves and to transfer to every object those qualities with which they are familiarly acquainted." Some such reflections, we feel, might have told as

truly the results in the experiment as the statistical record without the expenditure of time and labor.

It gives us a sort of what's-the-use feeling to contemplate such experiments and when it comes to being told that our friends judge us more accurately than ourselves, we like to turn back again to Browning and refresh our spirit with

> Now who shall arbitrate?
> Ten men love what I hate,
> Shun what I follow, slight what I receive;
> Ten, who in ears and eyes
> Match me: we all surmise,
> They, this thing, and I, that: whom shall my soul believe?
>
> Not on the vulgar mass
> Called "work," must sentence pass,
> Things done, that took the eye and had the price;
> O'er which, from level stand,
> The low world laid its hand,
> Found straightway to its mind, could value in a trice:
>
> But all, the world's coarse thumb
> And finger failed to plumb,
> So passed in making up the main account:
> All instincts immature,
> All purposes unsure,
> That weighed not as his work, yet swelled the man's amount:
>
> Thoughts hardly to be packed
> Into a narrow act,
> Fancies that broke through language and escaped:
> All I could never be,
> All men ignored in me,
> This I was worth to God, whose wheel the pitcher shaped.

Starting with the assumption that some qualities are more indicative of "General Intelligence" than others and more comprehensive, we venture to suggest the value of tests that might get at such information as

1. The normal size of the angle of the cone of vision within which perception is clear.

2. The normal region of awareness in general and for specific classes of stimuli.

3. The power to reach decisions or "make up one's mind."

4. That given by Spencer, "Study of Sociology," p. 32—

"If you want roughly to estimate any one's mental caliber, you cannot do it better than by observing the ratio of generalities to personalities in his talk—how far simple truths about individuals are replaced by truths abstracted from numerous experiences of men and things."

The promised remarks on educational scales must hold over, and preface the study of correlation, coefficients and ratios, correction formulas, the methods of ranks and grades and the "Order of Merit Method."

Summary.

The "first scientific attempt" by Mrs. Sophie Bryant, D.Sc. Lond., recognized matters of fundamental importance that, at least some, later workers would have done well to bear in mind.

The "movement against mathematics" that stirred up this long investigation, and the causes that influenced changes in curricula, were due in large part to the sanguine hopefulness, the exuberant suggestiveness of Prof. E. L. Thorndike and others who followed him, to the resultant impression and sometimes profession that certain matters had been scientifically established which in some cases are even less confidently affirmed to-day, and to the employment of the mathematical forms and methods of handling scientific data and the transfer to the work of the universal feeling of certitude or confidence in mathematics. Finally, this confidence was possibly strengthened for many by the refinements introduced by Dr. C. Spearman, of the University of London, the validity of which for most data to which they were in this field applied is, to say the least, yet to be proven.

Some search has not led us to disagree with Dr. Henry S. Pritchett in saying that there is "no science as yet of administration of educational affairs."

There have been tests that have in a very rough way given a first approximation to what may reasonably be expected in certain school grades in some of the school subjects. The "principle" of tests has possibly been vindicated as Stern has pointed out.

The great amount of time that has been spent on tests by those not scientifically qualified to administer them is deplorable. Qualified scientists employ enough time in arriving at the best without encouraging promiscuous testing by all sorts of novices.

The carelessness or indifference that permits stealthy implications in phraseology is also to be deplored.

Lastly, it is most earnestly recommended that all who have to teach mathematics make serious endeavor to get the spirit of the subject and a knowledge of those parts to which what they teach immediately leads so that thus you may be able to put to silence ill-founded attacks, and assist in spreading a better understanding of a science that can be made to induce to more general clear thinking on the every-day affairs of life.

COLUMBIA UNIVERSITY,
NEW YORK CITY.

NEW BOOKS.

The Human Worth of Rigorous Thinking. By CASSIUS J. KEYSER. New York: Columbia University Press. Pp. 314. $1.75 net.

These essays and addresses give a many-sided exposition of the spiritual significance of mathematics: the science which Plato called "divine," which Goethe called "an organ of the inner higher sense," which Novalis called "the life of the gods," and which Sylvester called "the music of reason." No appreciable demand is made upon the reader's knowledge of mathematical technique, the discussions being conducted in the language current among educated men and women. The book makes its appeal, not to technicians as such, but to the far wider circle of readers who are interested, whether as teachers or as students, in the solid things of literature, philosophy and science.

CONTENTS: The Human Significance of Mathematics. The Human Worth of Rigorous Thinking. The Humanization of the Teaching of Mathematics. The Walls of the World; or Concerning the Figure and the Dimensions of the Universe of Space. Mathematical Emancipations: Dimensionality and Hyperspace. The Universe and Beyond: the Existence of the Hypercosmic. The Axiom of Infinity: a New Presupposition of Thought. The Permanent Basis of a Liberal Education. Graduate Mathematical Instruction for Graduate Students not Intending to Become Mathematicians. The Source and Functions of a University. Research in American Universities. Principia Mathematica. Concerning Multiple Interpretations of Postulate Systems and the "Existence" of Hyperspace. Mathematical Productivity in the United States. Mathematics.

The author is a rigorous thinker himself and teachers of mathematics will do well to read the book.

Man Proposes. By ELIOT H. ROBINSON. Boston: The Page Company. Pp. 359. $1.25 net.

The romance of John Alden Shaw while at Newport on legal business is not only full of mystery but it contains a deep moral problem. The plot is rather complex and holds the interest of the reader to the end.

Our Little Saxon Cousin of Long Ago. By JULIA DARROW COWLES.

Our Little Viking Cousin of Long Ago. By CHARLES H. L. JOHNSON. Boston: The Page Company. 60 cents each.

These two new titles of the series will be welcomed by those who have read some of the earlier volumes. A very interesting account of the children of other lands.

Automobile Charts. By VICTOR W. PAGE. New York: The Norman W. Henley Publishing Co. 25 cents each.

Location of gasoline engine troubles made easy. Location of carburetion troubles made easy. Location of ignition system troubles made easy. Location of engine cooling and lubricating troubles made easy. Location of Ford engine troubles made easy. Lubrication of the motor car chassis.

Modern Starting, Lighting and Ignition Systems. By VICTOR W. PAGE. New York: The Norman W. Henley Publishing Company. Pp. 509. $1.50.

This practical volume has been written with special reference to the requirements of the non-technical reader desiring easily understood explanatory matter relating to all types of automobile ignition, starting and lighting systems. It can be understood by anyone, even without electrical knowledge, because elementary electrical principles are considered before any attempt is made to discuss features of the various systems. These basic principles are clearly stated and illustrated with simple diagrams. *All the leading systems of starting, lighting and ignition have been described and illustrated with the co-operation of the experts employed by the manufacturers. Wiring diagrams are shown in both technical and non-technical forms. All symbols are fully explained. This is a book of real merit.*

A comprehensive review of modern starting and ignition system practice, giving full instructions for the repair and care of storage batteries, generators, regulating devices, starting motors, etc., and all representative systems are described in detail, the text matter being accompanied by complete diagrams showing all connections and the relation the various parts of the assembly bear to each other. Complete data is given for locating troubles in all systems, the various steps being considered in a logical, systematic manner, that can be easily followed by those without expert electrical knowledge. All ignition systems receive full consideration, starting with the simplest battery and coil forms found on early cars to the modern short-contact timer and magneto methods used with the latest eight and twelve cylinder motors. Every ignition, starting or lighting system component is considered individually and full directions are given for making all repairs. This book is unusually complete as it also includes descriptions of various accessories operated by electric current, such as electrical gear shifts, brake actuation, signaling devices, vulcanizers, etc. Considers the systems of cars already in use as well as those that are to come in 1916. A book every one needs.

The Modern Gasoline Automobile. By VICTOR W. PAGE. New York: The Norman W. Henley Publishing Co. 1916 edition. Pp. 816. $2.50.

The most complete treatise on the gasoline automobile issued. Written in simple language by a recognized authority, familiar with every branch

of the automobile industry. Free from technical terms. Everything is explained so simply that anyone of ordinary intelligence may gain a comprehensive knowledge of the gasoline automobile. The information is up to date and includes, in addition to an exposition of principles of construction and description of all types of automobiles and their components, valuable money-saving hints on the care and operation of motor cars propelled by internal combustion engines.

The subject of electrical motor starting systems has been considered at length and all leading systems and their components described. A discussion on ball and roller bearing, their maintenance and installation, has also been included, and a number of other features of timely interest, such as latest types of gasoline and kerosene carburetors, cyclecar power plants, the Fischer slide valve motor, detachable wire wheels, cantilever springs, eight- and twelve-cylinder motors, new valve operating systems, Stewart-Warner vacuum fuel feed, boat type body design, leather universal joints, Entz electric transmission, positive differential, armored automobile, hydraulic brakes, etc.

Entirely new material has been added on tractors in three- and four-wheel forms, cyclecars and agricultural tractors or automobile plows; combination gasoline-electric drive, front-wheel and four-wheel drive and steer systems and other important developments in power-propelled vehicles. The discussion of power transmission methods has been augmented by consideration of the skew bevel gear and two-speed direct drive rear axle, as well as several new forms of worm gear drive, etc., have been added to bring the work thoroughly up to date.

The Spell of Egypt. By ARCHIE BELL. Boston: The Page Company. Pp. 366. $2.50 net.

Much has been written about Egypt and yet not so much but that much more is left to write about. It is a wonderful land for the traveler and archæologist, and one of never-failing interest to the general reader. This volume will be found not only entertaining but instructive as well as reliable in its information. It is well written and profusely illustrated.

How to Use Your Mind. By HARRY D. KITSON. Philadelphia: J. B. Lippincott Company. Pp. 216. $1.00 net.

As the author says in the preface "Educational leaders are becoming increasingly aware of the necessity for teaching students not only the subject matter of study but also methods of study." When teachers do this as they should there will be much less wasted time and much more real progress in our schools. To know how to work efficiently is half the battle and teachers will find this book very helpful in teaching how to study.

The Pioneer Boys of the Columbia. By HARRISON ADAMS. Boston: The Page Company. Pp. 345. $1.25 net.

This is a story of Dick and Roger Armstrong, who were sturdy pioneers of the great northwest. It contains much of valuable historical information and will stimulate interest in the early days of our country.

Eleanor of the Houseboat. By LOUISE M. BREITENBACH. Boston: The Page Company. Pp. 300. $???.

Anne's Wedding. By ISLA MAY MULLINS. Boston: The Page Company. Pp. 329. $1.25 net.

This new book contains the story of the Southern family which many read about in the preceding volumes, "The Blossom Shop" and "Anne of the Blossom Shop," of which this is a sequel. These stories, with their picturesque Southern setting, have a charm for old and young. There is very much of sunshine and cheerfulness throughout which makes life seem brighter.

Sylvia of the Hill Top. By MARGARET R. PIPER. Boston: The Page Company. Pp. 311. $1.25 net.

This is a sequel to "Sylvia's Experiment, The Cheerful Book," and is a joyous record of helpfulness, full of amusing incidents and abounds with good fellowship.

Men of the Old Stone Age. By HENRY FAIRFIELD OSBORN. New York: Charles Scribner's Sons. Pp. 545. $5.00 net.

The history of the country of the men of the Old Stone Age has been developed chiefly by French archeologists, while the more recent discoveries have been given by various authors, such as Lord Avebury, Professor Sollas, and Professor Obermaier. The author of this volume has made it his task to give a chronological treatment of the subject suitable for the general reader and to "connect the environment, the animal and human life, and the art." The book is splendidly made and well illustrated and the reader will find it very entertaining.

School Ethics. By ELEANOR MARCHBANKS. Boston: The Four Seas Company. Pp. 178. $1.25 net.

There is great need to-day of more effective ethical training in the schools, training, not from text-books, but by example and illustration on the part of the teacher. Many teachers pay no attention to it, thinking perhaps it is no part of their work and other teachers have not learned how to do it effectively.

One of the best books we have seen on the subject is this little volume by Miss Marchbanks. It describes how she does it and will be found very suggestive for others. As good moral character is the prime requisite for good citizenship, more teachers should make this an aim in

their school work, and the schools would be benefited by having more teachers of the type of this author.

Ruler and Compasses. By HILDA P. HUDSON. London: Longmans, Green & Co. Pp. 143.

Miss Hudson in this little volume starts with Euclid's three postulates and shows that the use of the ruler corresponds with the linear equations and the use of the compasses with the quadratic equation. The answer then to the question as to what constructions are possible with ruler only and what constructions with the ruler and compasses, which answer geometry failed to give, is furnished by analysis. In other words, those problems and those alone can be solved by ruler only, which can be made to depend on a linear equation; and those problems and those alone can be solved by ruler and compasses, which can be made to depend on an algebraic equation, whose degree is a power of 2 and whose roots can be found by rational operations and the extraction of square roots only. The book contains very much of interest and profit for teachers of geometry.

Differential and Integral Calculus. By CLYDE E. LOVE. New York: The Macmillan Company. Pp. 339. $2.10.

This appears a very good book on the subject, for the student using it should get a clear understanding of the various principles as he proceeds. The treatment of some topics would seem perhaps too abbreviated but brevity must be characteristic when the whole course, including three chapters on differential equations, is contained within the compass of 339 pages. The book is carefully written and appears to be well adapted for class use.

Quartic Surfaces. By C. M. JESSAP. Cambridge: The University Press. G. P. Putnam's Sons, American representatives. Pp. 198. $3.00.

The aim of the author in this volume is to give a brief account of the principal known properties of quartic surfaces possessing nodes or nodal curves. On account of Hudson's work on "Kummer's Quartic Surface" a treatment of that surface with its special forms is omitted here. Ruled quartic surfaces are also omitted.

The Introduction gives a brief summary of all the leading results discussed later in the volume. The chapter headings are as follows: I. Quartic Surfaces with Isolated Singular Points; II. Desmic Surfaces; III. Quartic Surfaces with a Double Conic; IV. Quartic Surfaces with a Nodal Conic and Additional Nodes; V. The Cyclide; VI. Surfaces with a Double Line, Plücker's Surface; VII. Quartic Surfaces with an Infinite Number of Conics; Steiner's Surface; The Quartic Monoid; VIII. The General Theory of Rational Quartic Surfaces; IX. Determinant Surfaces.

The Integration of Functions of a Single Variable. By G. H. HARDY. Cambridge: The University Press. G. P. Putnam's Sons, American representatives. Pp. 67. 75 cents.

This is a new edition of number 2 of the "Cambridge Tracts in Mathematics and Mathematical Physics," which came out some years ago and differs from the former edition chiefly in replacing a faulty proof of Abel's by another.

A Treatise on the Circle and Sphere. By JULIAN LOWELL COOLIDGE. Oxford: The Clarendon Press. Pp. 603. $6.75.

Every student of geometry will agree that the circle and sphere are perhaps the simplest of the geometrical figures, yet comparatively few realize how much there is centered about them. They force themselves on our attention in all parts of geometrical science, and have been treated by many prominent mathematicians like Steiner, Chasles, Möbius and others since the time the ancient Greeks did so much to complete the treatment of the circle. These figures in projective geometry, in function theory and in differential geometry have been treated by many authors, but Professor Coolidge has aimed in this volume "to present a consistent and systematic treatment of these various theories."

The magnitude of this undertaking will be better appreciated from the chapter headings, which are as follows: I. The Circle in Elementary Plane Geometry; II. The Circle in Cartesian Plane Geometry; III. Famous Problems in Construction; IV. The Tetracyclic Plane; V. The Sphere in Elementary Geometry; VI. The Sphere in Cartesian Geometry; VII. Pentaspherical Space; VIII. Circle Transformations; IX. Sphere Transformations; X. The Oriented Circle; XI. The Oriented Sphere; XII. Circles Orthogonal to one Sphere; XIII. Circles in Space, Algebraic Systems; XIV. The Oriented Circle in Space; XV. Differential Geometry of Circle Systems.

Even in a work of this magnitude some theorems must be omitted and the author gives preference to those which are most general in scope and to those which are unaltered by inversion. It is a piece of work which all students of higher geometry will want to read.

Second Year Mathematics. By ERNST R. BRESLICH. Chicago: The University of Chicago Press. Pp. 248+xviii. Price $1.00 net.

Mr. Breslich's "First Year Mathematics" was reviewed in the issue of March, 1916, and what was said of that book helps to explain the second volume. In this volume geometry in two and three dimensions becomes the principal subject, algebra and trigonometry being taken up where they are needed or fit in. The book has many excellent features and seems to be a fitting continuation of the earlier one. It is planned to be usable after first year algebra as well as following its own first volume.

American Private Schools. By PORTER E. SARGENT. Boston. Pp. 604.

This is the second annual edition of "The Best Private Schools" which was reviewed in the issue of December, 1915. The book has been greatly improved, and contains so much material of value that it should be in the hands of all school principals. It is proposed to make it a complete, definite encyclopedia of private schools, with a wealth of additional information along educational lines There are still errors and omissions, however, some of the officers of associations not having been changed since last year, and some of the most interesting of the newer schools being omitted.

Drill Book in Plane Geometry. By ROBERT R. GOFF. Boston: The Riverdale Press. Pp. 113 + vii.

This book is evidently the outgrowth of Mr. Goff's earlier one, "Syllabus of Plane Geometry Arranged for Emphasis and Method." It is a "syllabus method" text, and does not even include definitions or sample proofs in its order. It has, however, a dictionary of terms, and notes on the various topics. Mr. Goff has arranged the propositions in groups according to what is to be proved, and he emphasizes classification by use throughout. Practically all of the development in the book is by means of suggestive questions, and excellent summaries are given.

There is no doubt that such a text as this will give remarkably good results when used by a capable teacher, but at present most teachers need a little more direction than it gives. However, if it does no more than add another suggestion as to the value of heuristic methods it will prove worth while.

Plane and Solid Geometry. By WILLIAM BETZ and HARRISON E. WEBB. Boston: Ginn & Co. Pp. 507. Price $1.36.

The "Plane Geometry" by these authors was reviewed in an earlier number. This book combines the two geometries in a well-bound, compact volume. The solid geometry is a consistent continuation of the plane, with carefully worked out order, interesting applications, and good figures. It has many interesting details, such as the inclusion of both informal and formal methods of proof for some of the theorems, and the combination of cones and pyramids into one topic. The authors are progressive teachers and they have written a book that will prove interesting to teachers of geometry.

Drawing for Builders. By R. BURDETTE DALE. New York: John Wiley and Sons. Pp. 166 + v. Price $1.50 net.

This is an excellent addition to the increasing number of vocational texts. It serves the double purpose of preparing a student for more advanced work in architectural drawing, and helping the man already in practical work or planning to do such work without taking an architectural course. Much of the work can, if necessary, be done without

an instructor, although the book is designed for class use. It contains good chapters on drawing instruments and their use, lettering, and straight line projection, as well as the work on problems. The plates are $7 \times 8\frac{1}{4}$ inches and are of excellent quality.

Practical Short Methods in Rapid Calculation. By WILLIAM O. BELL. Kansas City: Burton Publishing Co. Pp. 118. Price $1.50.

This book takes up short methods of handling the fundamental operations and ordinary business processes. It goes into great detail in this field, giving many valuable methods. It seems, however, that some of the methods require too much remembering for their occasional use to pay, and that considerable classification and generalization should have been done in order to reduce the rules to a more useful minimum.

Combinatory Analysis. By P. A. MACMAHON. Cambridge: The University Press, G. P. Putnam's Sons, American Representatives. Vol. I, pp. 300, $4.50 net. Vol. II, pp. 340, $5.50 net.

The author of this work has for long years been much interested in the subject as evidenced by the many research papers that have come from his pen. His main object here has been to present a general doctrine, which is not only very general, but which connects up much that has seemed disconnected in the past. Some remarkable theorems are given by means of which many difficult problems are readily solved. The relation of the theory of combination to that of monomial symmetric functtions is given and use is made of the generating function of symmetric functions. The reader will find that the present work is much more general than Netto's "Combinatorik" and overlaps it very little. Here we have an account of what is in reality a new theory. The treatment is algebraic rather than arithmetic, and many investigations which are arithmetical are omitted on that account.

Volume I is divided into six sections: Symmetric Functions; Generalization of the theory of section I; Permutations; Theory of the compositions of numbers; Distribution upon a chess board to which is prefixed a chapter on perfect partitions; The enumeration of partitions of Multipartite numbers.

Volume II takes up the theory of the partitions of numbers, giving Euler's point of view and then a new method based upon the Diophantine inequalities which proves to be of great value in generalizing. It is divided into five sections: The partition of numbers; New basis of the theory of partitions; Partitions in two dimensions; Symmetric functions of several systems of quantities with some application to the distribution theory.

It is a splendid work by an able author.

How to Learn Easily. By G. VAN NESS DEARBORN. Boston: Little, Brown, and Company. Pp. 227. $1.00.

This is another timely book on this important subject of teaching students how to study. The chapter headings are: Economy in study; Ob-

servation and the taking of notes; Educative imagination; Books and their educative use; Is your "Thinker" in order? Examination preparedness.

Teachers will find here very many valuable hints, and a careful following of them should produce much inprovement.

Modern Business Arithmetic, Brief Course. By HARRY A. FINNEY and JOSEPH C. BROWN. New York: Henry Holt and Co. Pp. v + 298.

This is a practical course in the arithmetic actually used in business. All other matter has been eliminated, and the order of topics, discussions and drill are all based on business customs. The book is attractively gotten up, and contains many excellent features.

Introduction to Mathematics. By ROBERT L. SHORT and WILLIAM H. ELSON. Boston: D. C. Heath and Co. Pp. vii + 200.

This is an attempt to correlate arithmetic, algebra and geometry for a first-year course in high-school mathematics. The parts of these branches are not intermingled in as detailed a way as is done in some other such books, but follow each other in alternate sections of considerable length. The course covers algebra through fractional equations, and straight-line geometry (about sixty theorems) to proportion.

Plane Geometry. By FLETCHER DURELL and E. E. ARNOLD. New York: Charles E. Merrill and Co. Pp. 300. Price 88 cents.

This book follows the recommendations for minimum lists of theorems in its propositions, and makes some changes in order. The propositions are usually proved in full except for questions as to authorities, original work being confined to the exercises. There is some good training in methods of attack, and considerable excellent exercise material. The book has an attractive appearance, its pages being well made up and in clear type.

Laboratory Manual of Inorganic Chemistry for Colleges. By LYMAN C. NEWELL. Boston: D. C. Heath and Co. Pp. vi + 240. Price 60 cents.

This manual contains over 350 experiments chosen from those actually done by the author's classes in Boston University. They seem to cover the necessary topics quite thoroughly, and to be well adapted to college classes.

Number Stories. By ALHAMBRA G. DEMING. Chicago: Beckley Cardy Company. Pp. 205. Price 60 cents.

This book contains five very cleverly written stories in which problems are so interwoven as to be an integral part of the story and to add rather than detract from its interest The stories have the double purpose of teaching morals through stories, and presenting a varied list of arithmetic questions in a live setting.

NOTES AND NEWS.

The following names were accidentally omitted from the list of members published in the September issue:

Aiken, Carrie E., 111 E. 5th St., Jamestown, N. Y.
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Wolfenden, Emma, 5901 Ridge Ave., Roxborough, Philadelphia, Pa.
Zagat, Lillian, Washington Irving H. S., New York, N. Y.

New Members.

Swenthal, Murray J., 184 S. 2d St., Brooklyn, N. Y.
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McCardell, J. LeRoy, 261 S. Potomac St., Hagerstown, Md.
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Holden, M. Elizabeth, 616 N. Arlington Ave., Baltimore, Md.
Mills, Ruth B., Chesapeake City, Md.

Colliton, Mr. J. W., 731 Monmouth St., Trenton, N. J.
Scobey, Frank H., State Normal School, Trenton, N. J.
Fowler, Miss Laura, 329 Dolphin St., Baltimore, Md.
White, Annabel Lee, 2446 Maryland Ave., Baltimore, Md.
Wilhelm, William H., Elderslie Ave., Mt. Washington, Md.
Belcher, Donald Ray, 2940 Broadway, New York, N. Y.
Hance, William, 249 Mt. Prospect Ave., Newark, N. J.
Talbot, Miss A. May, 452 Greenwood Ave., Richmond Hill, N. Y.
Putney, Miss E. N., 45 Cook Ave., Elmhurst, N. Y.
Briggs, Eva E., 92 Gates Ave., Brooklyn, N. Y.
Gordon, Louis, 728 Annsbury St., Philadelphia, Pa.

THE twenty-seventh meeting of the Association was called to order by the president in the chemistry lecture room, Catherine Hooper Hall, Goucher College, Baltimore, Md., Saturday, December 2, 1916, 10:15 a. m. After a short address by the president the morning program was started. Mr. Colliton, of Trenton High School, gave an interesting talk on Students' Mathematics Clubs as conducted in his school. Miss Walker read a very interesting paper by Miss Webster, of Hunter College, on the same topic. A discussion followed by Dr. Hallett, Dr. Hawkes and Professor Requa.

Mr. Koch, of the School of Commerce, gave an interesting talk and demonstration on School Contests. Miss Wood, of William Penn High School, gave an interesting talk on the same topic, telling of some of the contests held in that school.

Professor Babb, of the University of Pennsylvania, then gave a short talk on Individual Recreations.

This part of the program was followed by the formation of the Southern Section, after which Mr. Koch, chairman of the Nominating Committee, conducted the election of officers of the General Association. The following officers were elected: President, Dr. Jonathan T. Rorer, William Penn High School, Philadelphia, Pa.; Vice President, Prof. Emma M. Requa, Hunter College, N. Y. City; Secretary, Mr. F. Eugene Seymour, State Normal School, Trenton, N. J.; Treasurer, Dr. Edw. D. Fitch, 4035 Locust Street, Philadelphia, Pa.; Council Member, Miss Grace S. Barker, The Baldwin School, Bryn Mawr, Pa.

The afternoon meeting was divided into two parts: (1) A round-table discussion of the topic "The Order of Teaching the Parts of the Calculus," presided over by Professor Bacon, of Goucher College, the leaders being: J. T. Lammond, Pennsylvania College, C. C. Grove, Columbia University, Walter Marriott, Swarthmore College, George H. Hallett, University of Pennsylvania. An interesting discussion followed these speakers. (2) A round-table discussion of the topic "Should Arithmetic be Taught to all Pupils of the High School? When? How much Time should be given to it?" This meeting was presided over by Dr. Edward D. Fitch, Episcopal Academy, the leaders being: Ruth Munhall, Germantown High School, Frank H. Scobey, State Normal School, Trenton, Elizabeth A. Hallock, Palmyra High School, Amy L. Clapp, South Philadelphia High School. An interesting discussion followed these speakers.

Both the morning and the afternoon meetings were largely attended.

THE New York Section of the Association of Teachers of Mathematics in the Middle States and Maryland met at Hunter College, Friday, November 24, 1916, with the following program: "The Contributions of Mathematics in the Development of Thought"—(1) Professor Charles C. Grove, "How to make the Possible Contributions Effective." (2) Dr. John L. Tildsley, "The consideration of mathematics as a vehicle and instrument of thought." (3) Mr. John F. Waters, "The common sense and the acquired sense of mathematics in the grades." (4) General Discussion, Leaders: Prof. Elizabeth B. Cowley, Mr. Herman H. Wright.

put aside for it. You can bring that same sense of delight into any home by sending *The Youth's Companion* to it for a year—$2.00—only four cents a week.

The Companion Home Calendar goes to every new subscriber and to everyone who makes a gift subscription.

You can, if you wish, take advantage of *The Companion's* special arrangement with *McCall's Magazine,* and by sending $2.10 get both publications for 1917. This makes two Christmas presents for the price of one. Everyone taking advantage of this offer has the choice of a free dress pattern sent by *McCall's Magazine* on receipt of a 2-cent stamp. *The Youth's Companion,* St. Paul St., Boston, Mass.

The Century for 1917 will contain an important series of articles on "The Reconstruction of Europe" by Herbert Adams Gibbons, author of "The New Map of Europe," etc., covering the tremendous and inescapable problems which Europe—and America too—must presently face. Dr. Gibbons does not speculate or prophesy, but upon wide and authoritative information

he brings to bear that wisdom which is the secret of prophecy. Each article takes up a specific case which illustrates one of the general broader problems of reconstruction. The scheme of articles is as follows: (1) "The Future of Poland" (the problem of the "buffer state" and of the separation of territories from their present owners). (2) "Great Britain in the Sudan" (the problem of European colonization and European rivalry in Africa.) (3) "What Confronts France" (the problem of social and economic rehabilitation). (4) "Italy in the Adriatic" (the problems of the peninsular power, irredentism and international alliances). (5) "The Balance of Power in the Balkans" (the whole Balkan tangle and the problem of Constantinople). (6) "The Arabian Khalifate" (the problem of Islam in the New Era, of Egypt and Turkey). Whether America takes part in the settlement or not, we shall be involved in it; and the more complete our understanding of it, the better for us and our country. There is no surer way of grasping these problems than by subscribing to *The Century* and following Mr. Gibbons's articles.